Autofocusing with the help of the **empirical Haar transform**

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- Model and formal assumptions

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- Generic algorithm and its properties

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A proper and reliable focusing algorithm is a conditio sine qua non of a 'good image'. Not only from an aesthetic vantage point, but also in automated applications.

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  - *information theory*, *probability theory* and *mathematical statistics*, as well.

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Solution

Our algorithm works with standard matrix sensors & standard optics, and employs standard transforms and routines...

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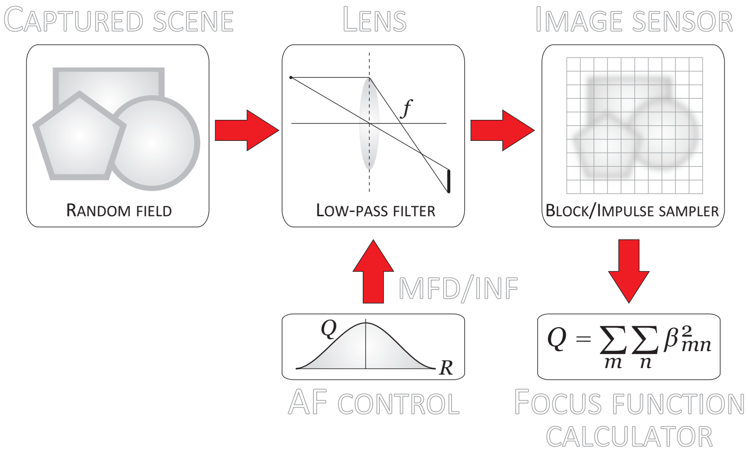
AF model



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3. Make it **reliable** in *noisy environments*!

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3. The **image sensor** acts as a *block sampler*, that the lens-produced image is *orthogonally projected* onto the space of piecewise constant functions.

AF algorithm foundations

- The AF algorithm is based on the following lemmas:

Lemma

Under assumptions **1-3**, *the variance of the captured image is a* **unimodal** *function w.r.t. the order of the lens filter and attains its* **maximum** *value for the in-focus image.*

Lemma

The variance estimate is **tantamount** *to the orthogonal expansion of the image acquired by the sensor.*

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  - polynomial, *e.g.* *Chebyshev*, *Legendre*, *Zernike* (in general—any *'people's polynomials'*).

Possible AF function computation implementations

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We can apply standard algorithms to find the **function's maximum** in a **noisy environment**:

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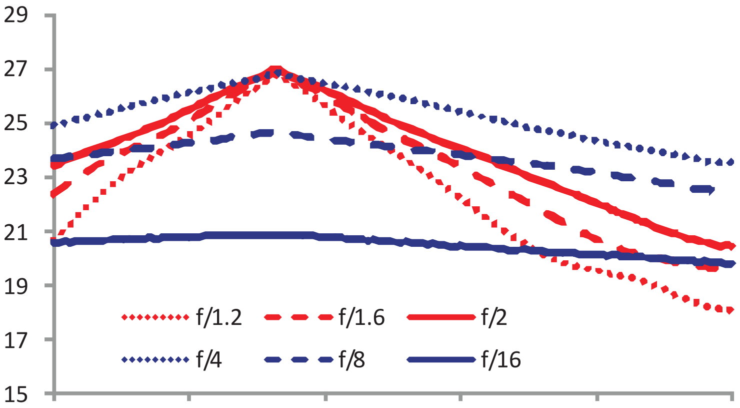
8. **Fast implementation** – all algorithms exploit '*fast*' transforms

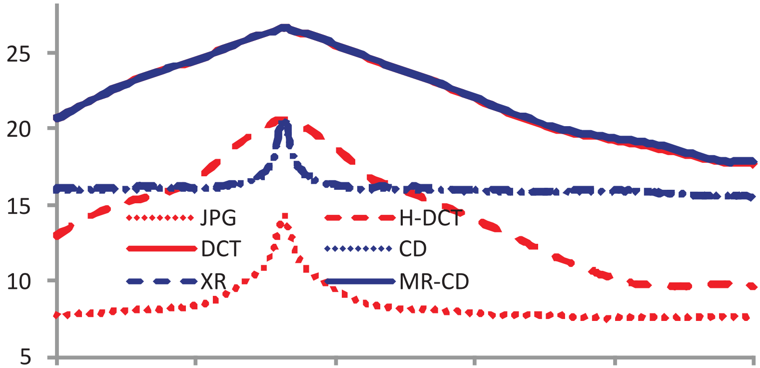
Experimental results

AF against aperture



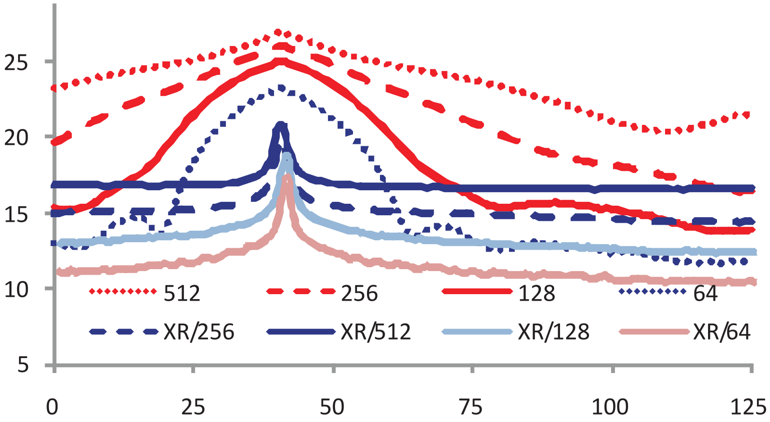
Experimental results

AF against transform coder



Experimental results

AF against image size



Single-Photon AF

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Can the generic algorithm be adapted to the Single-Photon Imagery?

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- For the others, it is just convenient to model them as a noise. . .

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- The basic transform step becomes a little bit more complicated than

$$\hat{\alpha}_{m-1,n} = \frac{\sqrt{2}}{2}\hat{\alpha}_{m,2n} + \frac{\sqrt{2}}{2}\hat{\alpha}_{m,2n+1}$$

vs.

$$\bar{\alpha}_{m-1,n} = \sqrt{\frac{I_{m,2n}}{I_{m-1,n}}}\bar{\alpha}_{m,2n} + \sqrt{\frac{I_{m,2n+1}}{I_{m-1,n}}}\bar{\alpha}_{m,2n+1},$$

with $I_{m-1,n} = I_{m,2n+1} + I_{m,2n+1}$ (where $I_{m,2n+1}, I_{m-1,n}$ are the *non-uniformity indices*).

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- But. . . requires computing **square roots**. . .

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- The rest of the **AF** algorithm remains unchanged.

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- Can *effectively* be implemented (*e.g. in situ*).

Example

The demonstration movie can be found at:

`http://diuna.iiar.pwr.wroc.pl/sliwinski/gss-af.avi`